

# PHYS 232

## The Stirling Cycle

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### Introduction

In this experiment you will study the Stirling cycle using a working Stirling engine. The Stirling engine that you will use is a “displacer-type” engine made by a company called Kontax Stirling Engines. Although this engine has been designed and marketed as a demonstration Stirling engine, with only minor modifications, we will use them to make some quantitative measurements.

Figure 1 shows the basic working principles of an ideal heat engine. Heat  $Q_H$  extracted from a hot thermal reservoir at temperature  $T_H$  is partially converted into useful work  $W$ . Excess heat  $Q_C$  is deposited into a cold thermal reservoir at temperature  $T_C$ .

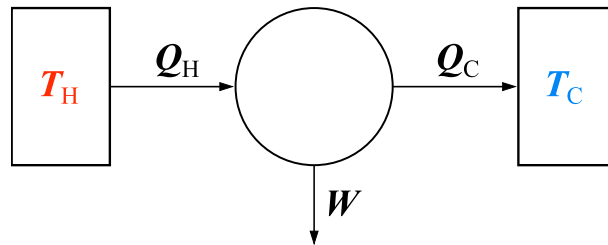


Figure 1: In a heat engine, heat  $Q_H$  is extracted from a hot thermal reservoir and heat  $Q_C$  is dumped into a cold thermal reservoir.  $W$  represents the useful work extracted from the cycle. Figure obtained and adapted from Wikimedia Commons.

The idealized Stirling cycle is shown graphically in the  $P$  versus  $V$  plot, known as a  $P$ - $V$  diagram, of Fig. 2.

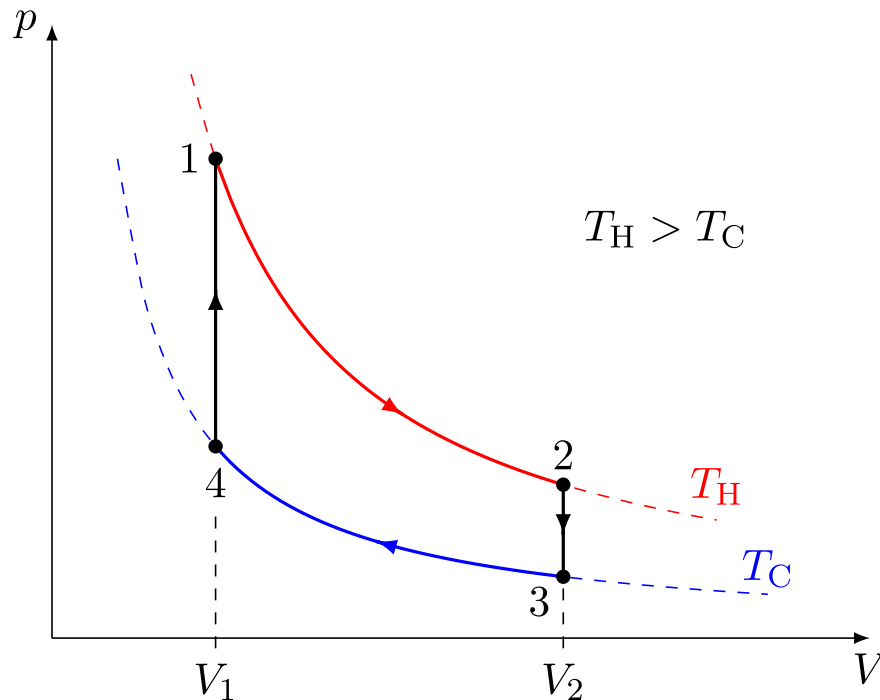


Figure 2: The Stirling cycle  $P$ - $V$  diagram. The enclosed area represents the work done by the cycle. Figure obtained and adapted from Wikimedia Commons.

$P$ - $V$  diagrams are a useful way to visualize the cycle of a heat engine because the area enclosed by the closed loop ( $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ ) is a measure of the work  $W$  done during each cycle. If the time to complete a single cycle (i.e. the period) is  $T$ , then the power output of the engine is given by  $P = W/T$ . On day one of this experiment, you will experimentally measure the  $P$ - $V$  diagram of a working Stirling engine. The data will be analyzed to determine the power output of the engine. On day two, you will run the Stirling cycle in reverse. In this case, instead of converting a temperature difference to mechanical work (as in a heat engine), you will convert mechanical work into a small, but measurable, temperature difference (refrigeration).

**Pre-lab Assignment**

*Question 1:* Suppose a gas is expanded in order to push a cylindrical piston of cross-sectional area  $A$ . Show that the work  $dW$  done by the gas to push the piston an infinitesimal distance  $dz$  is given by  $dW = P dV$  where  $P$  is the gas pressure and  $dV = A dz$  is the change in volume of the gas.

*Question 2:* In the Stirling cycle, steps 2 and 4 in the cycle of Fig. 2 are isochoric, or constant volume, processes. How much work is done by the gas during these steps?

*Question 3:* In the Stirling cycle, steps 1 and 3 in the cycle of Fig. 2 are isothermal, or constant temperature, processes. How much work is done by the gas during these steps? Suppose that gas temperature during step 1 is  $T_H$  and during step 3 it is  $T_C$ . Suppose also that the minimum and maximum gas volumes are  $V_1$  and  $V_2$ , respectively.

*Question 4:* Finally, what is the net work done by the gas after completing one full cycle?

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*Day 1*

In order to construct a  $P$ - $V$  diagram, then pressure and volume of the gas enclosed within the engine must be measured. Ultimately, we will only really be interested in how the pressure and volume of the gas *change* throughout the cycle. Figure 3 show a photograph of the actual Stirling engine that will be used for this experiment.

A hole has been drilled through the top plate of the engine. A differential pressure sensor is used to monitor the pressure of the gas contained within the engine relative to the external room pressure. The pressure sensor is powered with 5 V and it outputs a voltage that is proportional to the pressure difference. The output of the pressure sensor is processed electronically to establish a calibration factor of 1 kPa/V for the final output voltage representing the measured pressure difference. A negative (positive) voltage corresponds to an internal gas pressure that is less (greater) than the external room pressure.

The instantaneous volume of the gas in the engine is determine by the position of a graphite piston (see Fig. 3). The motion of the piston is synchronized to the orientation of the flywheel using a connecting rod. The volume of the gas is indirectly measured by tracking the orientation of the flywheel using optical choppers and photogates. The photogates are made using a light-emitting diodes (LEDs) and phototransistors. The states of the phototransistors are determined

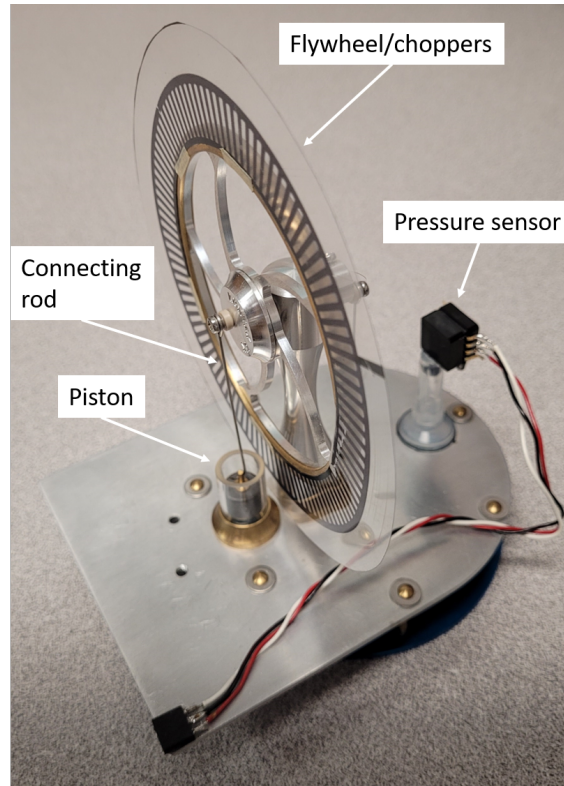


Figure 3: Photograph of the Stirling engine used in this lab. The engine is instrumented with a pressure sensor and a “chopper” that has been attached to the flywheel. The engine also has an internal Styrofoam “displacer” which is not shown in the figure.

by whether or not they are receiving light from the LEDs. If a chopper is blocking the light, then the phototransistor is not conducting and does not allow current to pass. On the other hand, when the chopper allows the light to pass through, the phototransistor does conduct current. In this experiment, we will use two choppers (an inner chopper and an outer chopper) and a pair of photogates. The outer chopper is entirely transparent except for a single thin black strip that is used to momentarily block the light. This single black strip is positioned such that when it is at the top of its rotation, the piston is at its lowest position. Thus, when the outer phototransistor is not conducting, we know that the gas volume is at its minimum.

The inner chopper has 100 equally-spaced black sections. By counting the number of times light to the inner phototransistor has been blocked, the orientation of the flywheel can be tracked with a resolution of  $3.6^\circ$ . Finally, from the piston-connecting rod geometry, the orientation of the flywheel can be used to determine the position of the piston and, hence, the change in volume of the gas inside the Stirling engine. The entire process of monitoring the photogates and calculating gas volume is automated electronically. The calibration factor used to convert the final output voltage representing the volume change is  $0.1 \text{ cm}^3/\text{V}$ .

## Procedure

1. Use the kettle to boil water.
2. Fill the coffee cup with hot water (right to the top) and place the Stirling engine on top of the cup.
3. Position the photogates such that the dark sections of the choppers will block the light from the LEDs.
4. Use your finger to give the flywheel a gentle “kick” start.
5. Connect the chopper and pressure sensor cables to the electronics box and connect the volume and pressure outputs to channels 1 and 2 of an oscilloscope, respectively. The electronics box also requires connections to  $\pm 15$  V and common (0 V).
6. Observe and comment on the volume and pressure oscillations.

Next, change the oscilloscope to the so-called “*xy* mode” which is used to display CH2 as a function of CH1 on the screen. Do this by first pressing the “Display” button and then changing the “Format” option from “YT” to “XY”. You should now be looking at a real-time measurement of the Stirling engine’s  $P$ - $V$  diagram. First, observe the  $P$ - $V$  diagram as the flywheel is spinning freely. Next, use the retort stand holding a piece of foam to apply light friction to the brass ring of the flywheel. The friction should cause the flywheel to spin more slowly, but not stop it completely. Observe the changes in the  $P$ - $V$  diagram.

To make some quantitative measurements, put the oscilloscope back into YT mode so that you’re observing the volume and pressure oscillations on the screen. Next, connect the oscilloscope to the lab computer using a USB cable and download the required LabVIEW program from the course website. Use the program to record the oscilloscope data to file. Record data both with and without friction applied to the flywheel. It would be a good idea to fill the cup with freshly-boiled water before acquiring the quantitative data. There is a Jupyter notebook (.ipynb file) on the course website that can be used to (1) plot the  $P$ - $V$  diagram using the measured pressure and volume oscillations and (2) determine the enclosed area of the  $P$ - $V$  diagram. Determine the work and power output of the engine both with and without friction applied to the flywheel.

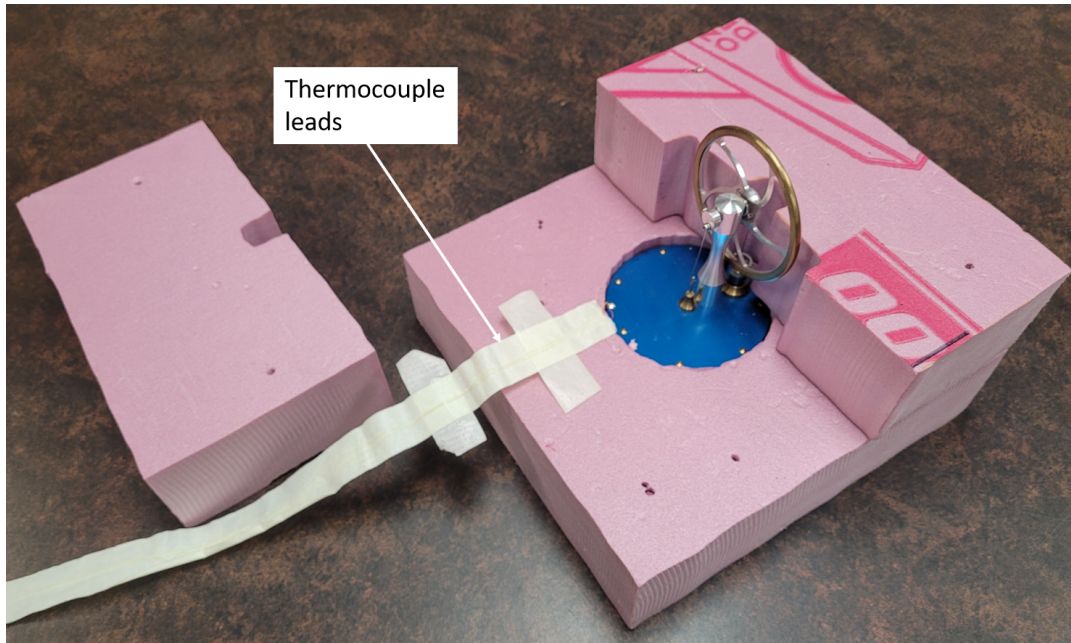


Figure 4: Setup used to operate the Stirling engine as a refrigerator. One of the Styrofoam blocks has been removed to expose the Stirling engine and thermocouple leads. The Dremel tool used to spin the flywheel is not shown.

### *Day 2*

To operate the Stirling engine as a refrigerator, we need to supply mechanical energy to spin the flywheel in order to establish a temperature difference across the top and bottom plates of the engine. The temperature difference will be measured using a type-K thermocouple. Note that, for type-K thermocouples, the lead with yellow insulation is the positive lead and the one with red insulation is the negative lead. The table needed to convert the measured thermocouple voltages to temperature is provided on the course website.

To spin the flywheel, a Dremel (rotary) tool with a felt attachment will be used. The Dremel tool spins too fast for our application. To slow it down, the tool will be plugged into a variac which will allow us adjust the supply voltage as a way to control the speed. We have found that the optimal rotational speed for this part of the experiment is 1 to 2 Hz. The temperature difference that will be established will be very small. In order to make a reliable measurement of the small temperature difference, the top and bottom plates of the Stirling engine need to be isolated from the room. As shown in Fig. 4, this isolation is achieved using Styrofoam blocks.

**Procedure**

1. Completely enclose the Stirling engine within the Styrofoam blocks.
2. Connect the thermocouple leads to a Keysight/Agilent/HP 34401A digital multimeter.
3. Record the multimeter readings using a second LabVIEW program that is available on the course website.
4. Use the Dremel tool and variac to spin the Stirling engine's flywheel at a rate of 1 to 2 Hz.
5. Watch to see the resulting temperature change.
6. Once the temperature difference has reached an equilibrium, try reversing the direction of the flywheel's rotation and continue to observe the temperature difference using the LabVIEW program.
7. Convert the recorded thermocouple voltages to temperature differences in units of degrees Celsius.